LNE
Le progrès, une passion à partager

& Mesures Références

Clés de la COMPÉTITIVITÉ
et d'un MONDE PLUS SÛR

IND03 HighPRES
High pressure metrology for industrial applications.

WP1 - Task 1.1.1 elasto-plastic strain

Thierry RABAULT (LNE – France)

Summary

1. Presentation
2. What is Plasticity ?
3. Plasticity criteria
4. Analytical model
5. Numerical study
6. Conclusion

Presentation

JRP IND03 = Study of Standard pressure 1,6 GPa ⇨ Plasticity

Generaly, Metrology = Plastic Domains

⇨No experience of LNE in this application field

Finite element software = Black box

Incorrect settings = Bad results

⇨Methodology to validate the results

Comparison FEM with analytical solutions

[illegible] project mi oct. 2011, 1st results end dec. 2011

What is Plasticity ?

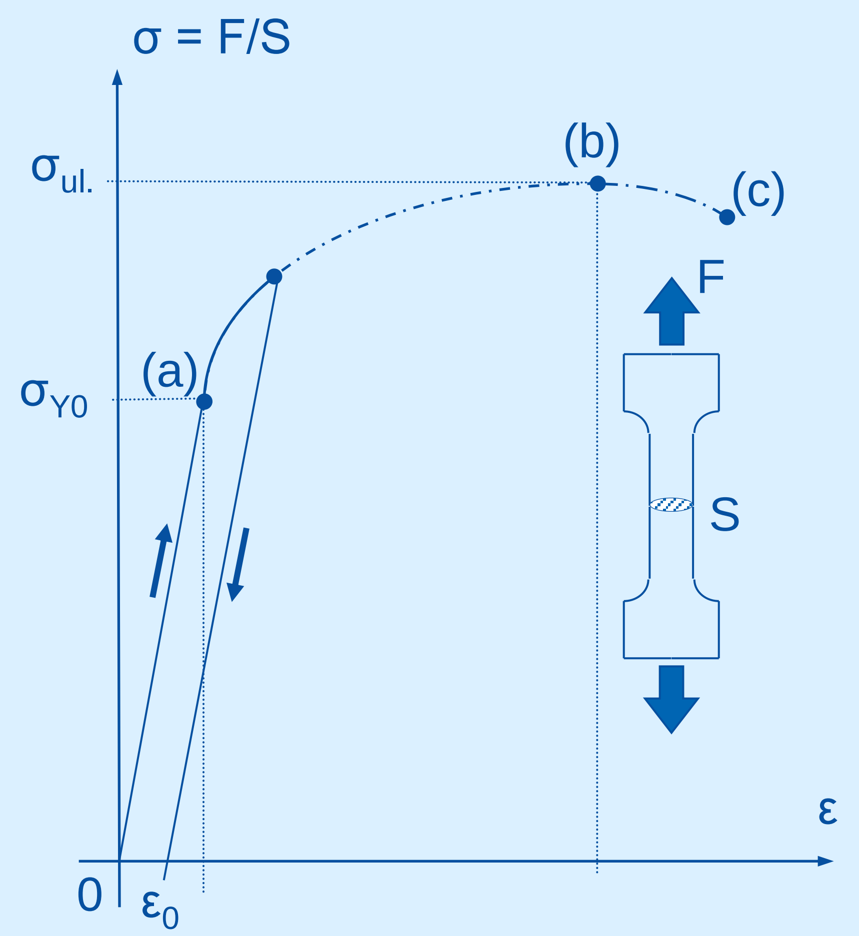
The mechanical properties of materials are determined from simple tensile tests on specimens.

3 domains :

- Elastic (0→a) linear
  - $\sigma_I = E.\varepsilon_I$
  - $\varepsilon_{II} = -\nu.\varepsilon_I$
- Plastic (a→b)
- Damage (b→c)

And finally

- Rupture (c)



Plasticity criteria

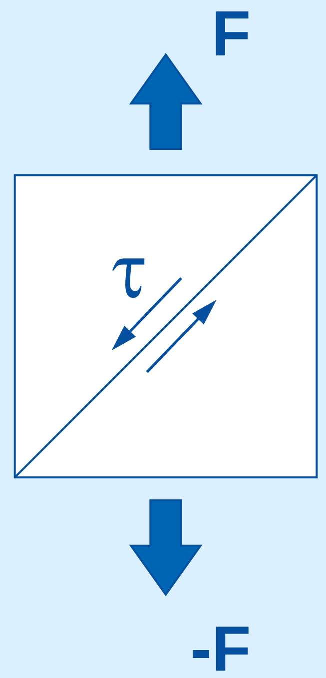
uniaxial tensile loading = plastic deformation by shearing

Tresca yield criterion (maximum shear stress criterion)

$$| \sigma_r - \sigma_\theta | = \sigma_{Y0}$$

if $| \sigma_r - \sigma_\theta | < \sigma_{Y0}$ = elastic domain

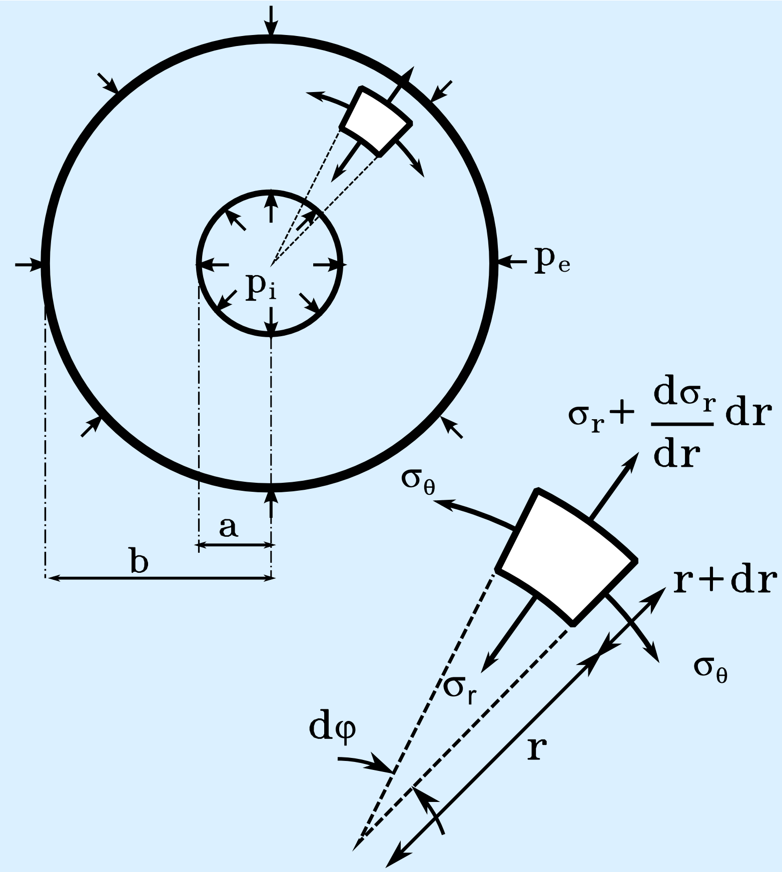
if $| \sigma_r - \sigma_\theta | > \sigma_{Y0}$ = plastic domain



Analytical model

Equilibrium equation :

$$\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0$$



$$\sigma_r \, r d\varphi + \sigma_\theta \, dr d\varphi - \left(\sigma_r + \frac{d\sigma_r}{dr} dr \right)(r + dr) d\varphi = 0$$

Analytical model

Equilibrium equation :

$$\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0$$

From the equilibrium equation, Hooke's law and the compatibility condition, we find the following equation :

$$\frac{d}{dr}(\sigma_\theta - \sigma_r) + 2\frac{\sigma_\theta - \sigma_r}{r} = 0$$

Whose general solution is :

$$\sigma_r = A - \frac{B}{r^2} \quad , \quad \sigma_\theta = A + \frac{B}{r^2}$$

Hooke's law :

$$\begin{cases} \varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] \\ \varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] \\ \varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)] \end{cases}$$

Radial strain : $\varepsilon_r = du/dr$

tangential strain : $\varepsilon_\theta = u/r$

Compatibility condition : $\varepsilon_r = \frac{d}{dr}(r\varepsilon_\theta)$

Analytical model

Equilibrium equation : $\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0$

From the equilibrium equation, Hooke's law and the compatibility condition, we find the following equation :

$$\frac{d}{dr}(\sigma_\theta - \sigma_r) + 2\frac{\sigma_\theta - \sigma_r}{r} = 0$$

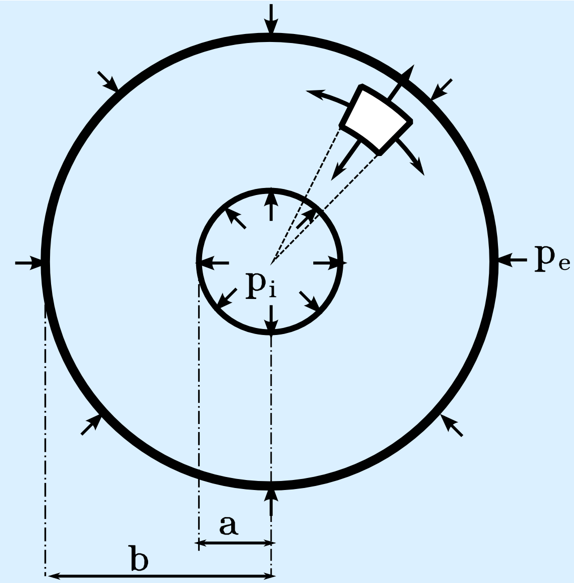
Whose general solution is :

$$\sigma_r = A - \frac{B}{r^2} \quad , \quad \sigma_\theta = A + \frac{B}{r^2}$$

Radial and tangential stress :

$$\sigma_r = -\frac{p}{(b/a)^2 - 1}\left[\frac{b^2}{r^2} - 1\right]$$

$$\sigma_\theta = \frac{p}{(b/a)^2 - 1}\left[\frac{b^2}{r^2} + 1\right]$$



Degrees of freedom :

$$\sigma_r\big|_{r=a} = -p \qquad \text{et} \qquad \sigma_r\big|_{r=b} = 0$$

Constraints A et B :

$$A = \frac{p}{(b/a)^2 - 1}$$

$$B = \frac{pb^2}{(b/a)^2 - 1}$$

Analytical model

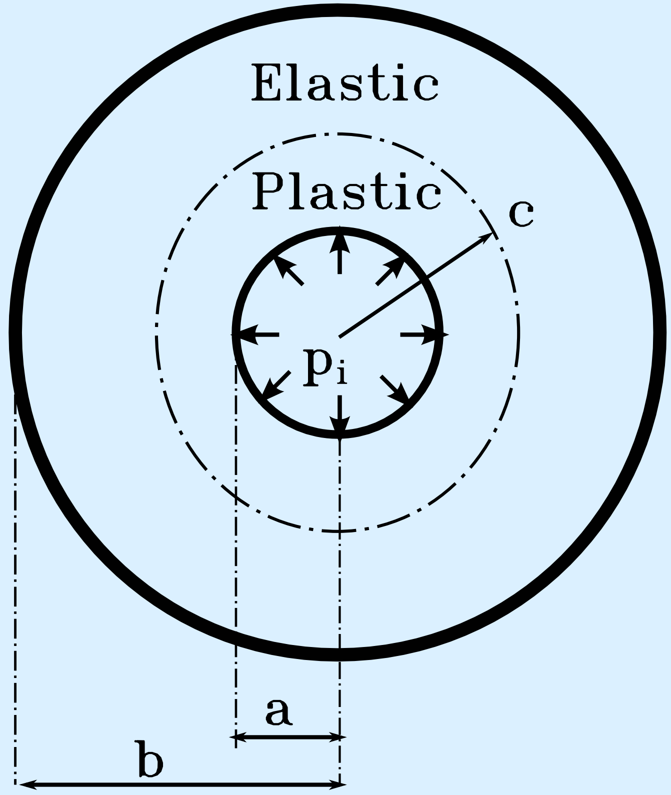
$$\sigma_r = -\frac{p}{(b/a)^2 - 1}\left[\frac{b^2}{r^2} - 1\right]$$

$$\sigma_\theta = \frac{p}{(b/a)^2 - 1}\left[\frac{b^2}{r^2} + 1\right]$$

+ Tresca criterion : $\sigma_\theta - \sigma_r = 2k$

= minimal pressure of plasticity

$$P_E = k\left(1 - \frac{a^2}{b^2}\right)$$



Analytical model

- Elastic domain for r > c

$$\sigma_r = -\frac{p_c}{(b/c)^2 - 1}\left[\frac{b^2}{r^2} - 1\right]$$

$$\sigma_\theta = \frac{p_c}{(b/c)^2 - 1}\left[\frac{b^2}{r^2} + 1\right]$$

- for r = c, limit between elastic and plastic domains

cisaillement maximum ⇨ Tresca criterion, $\sigma_\theta - \sigma_r = 2k$

for r = c , $$p_c = k\left(1 - \frac{c^2}{b^2}\right)$$

- Plastic domain for r < c

$$\sigma_r = -k\left[1 - \frac{c^2}{b^2} + \ln\left(\frac{c}{r}\right)^2\right]$$

$$\sigma_\theta = k\left[1 + \frac{c^2}{b^2} - \ln\left(\frac{c}{r}\right)^2\right]$$

Equilibrium equation $\sigma_\theta - \sigma_r - r\frac{d\sigma_r}{dr} = 0$

$$\frac{d\sigma_r}{dr} = \frac{2k}{r}$$

Solution $\sigma_r = 2k \ln r + C$

Degrees of freedom $\sigma_r\big|_{r=a} = -p$

$$C = -p - 2k \ln a$$

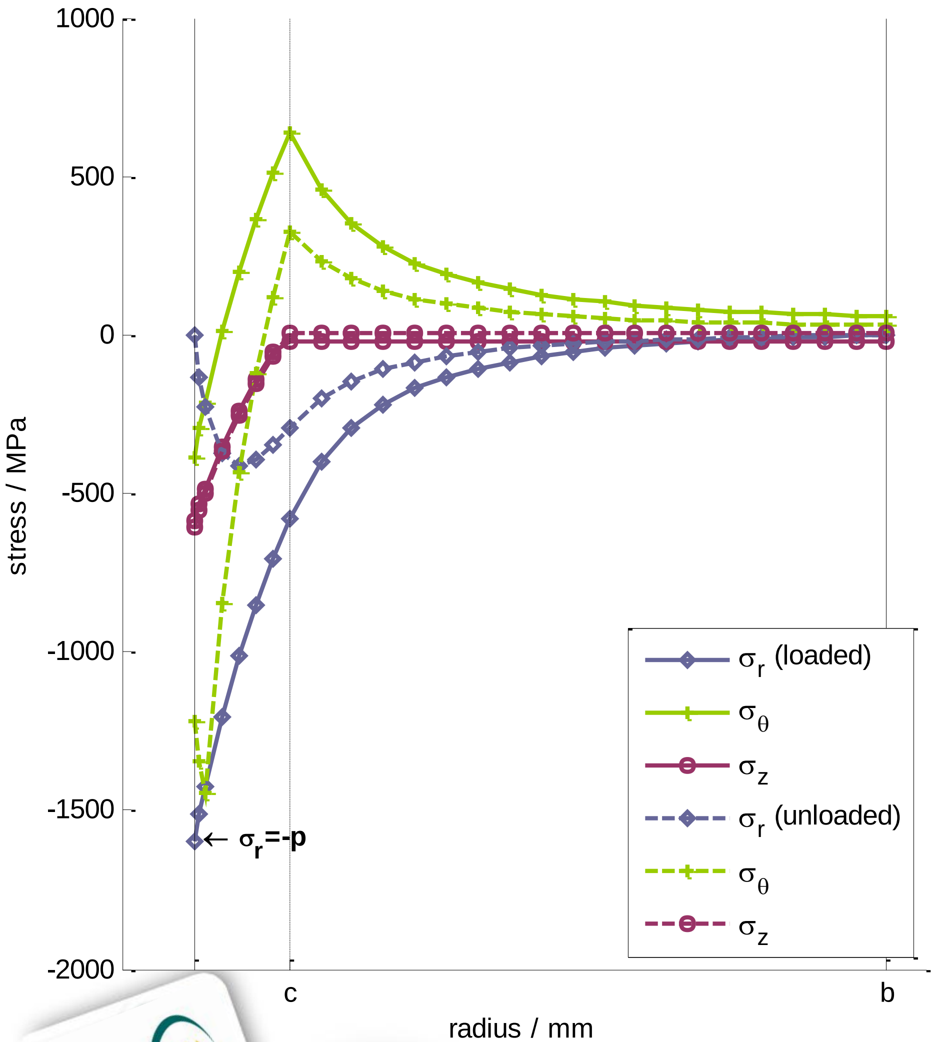
$$\sigma_r = -p + k \ln\left(\frac{r}{a}\right)^2$$

Continuity on c : $\sigma_r\big|_{r=c} = -p_c$

$$p = k\left[1 - \frac{c^2}{b^2} + \ln\left(\frac{c}{a}\right)^2\right]$$

Allows to determine the radius c.

Analytical model



	Loaded	Unloaded
plastic	$\sigma_r = -k\left[1-\frac{c^2}{b^2}+\ln\left(\frac{c^2}{r^2}\right)\right]$ $\sigma_\theta = k\left[1+\frac{c^2}{b^2}-\ln\left(\frac{c^2}{r^2}\right)\right]$	$\sigma_r = -k\left[\frac{p^2}{p_E^2}\left(1-\frac{a^2}{r^2}\right)+\ln\frac{r^2}{a^2}\right]$ $\sigma_\theta = -k\left[\frac{p}{p_E}\left(1+\frac{a^2}{r^2}\right)+\ln\frac{r^2}{a^2}-2\right]$
elastic	$\sigma_r = -k\left[\frac{c^2}{r^2}-\frac{c^2}{b^2}\right]$ $\sigma_\theta = k\left[\frac{c^2}{r^2}+\frac{c^2}{b^2}\right]$	$\sigma_r = -k\left(\frac{c^2}{a^2}-\frac{p}{p_E}\right)\left(\frac{a^2}{r^2}-\frac{a^2}{b^2}\right)$ $\sigma_\theta = k\left(\frac{c^2}{a^2}-\frac{p}{p_E}\right)\left(\frac{a^2}{r^2}+\frac{a^2}{b^2}\right)$

$$\sigma_z = \frac{\sigma_r+\sigma_\theta}{2} \qquad k = \frac{\sigma_{Y0}}{\sqrt{3}}$$

$$p = k\left[1-\frac{c^2}{b^2}+\log\left(\frac{c}{a}\right)^2\right] \qquad P_E = k\frac{1-(a/b)^2}{\sqrt{1+\frac{1}{3}(1-2\nu)^2(a/b)^4}}$$

Numerical study

Matlab + SdTools : No calculation of plasticity

⇨Purchasing the software ANSYS

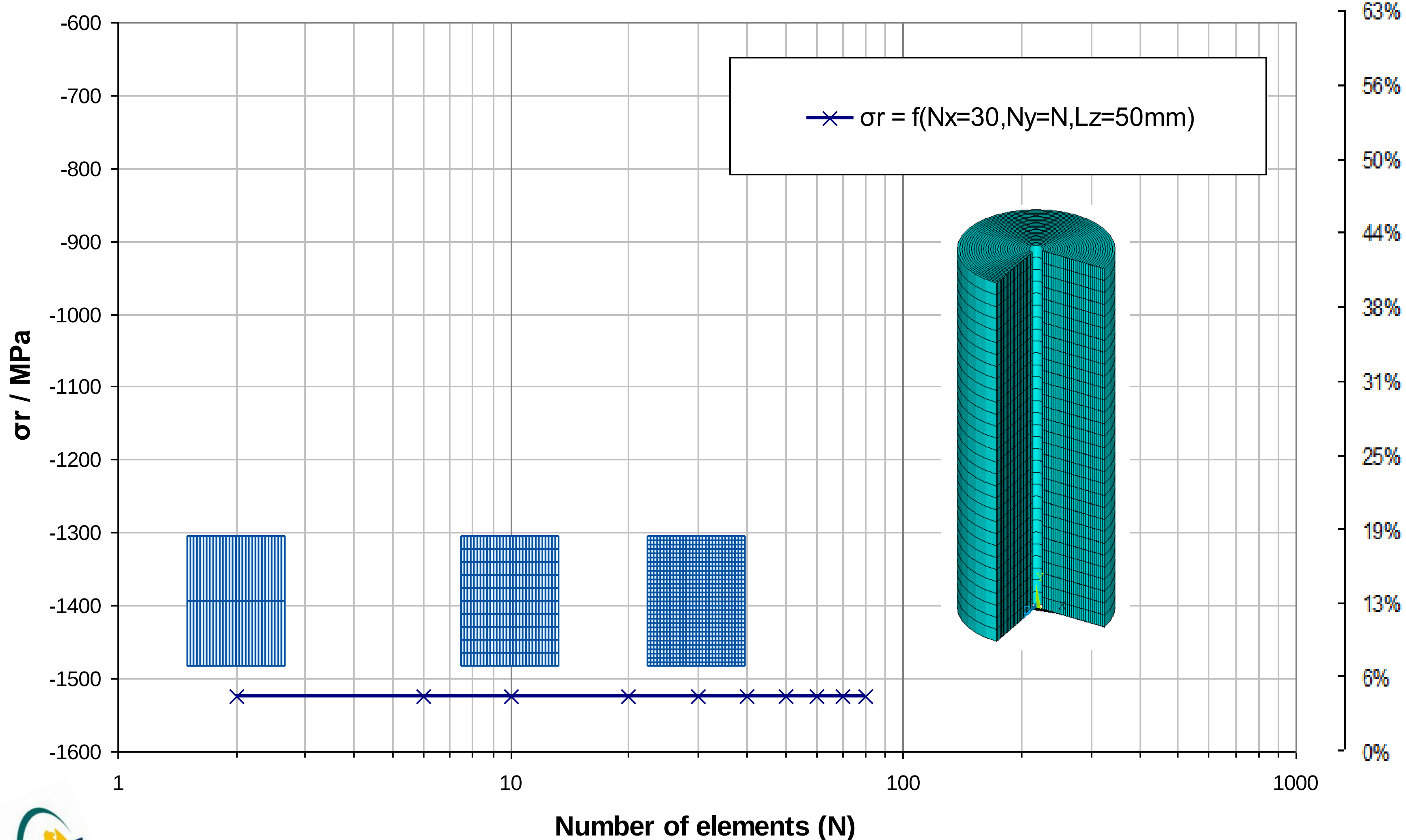
Workbench (graphical programmation)

Classic (APDL programmation *)

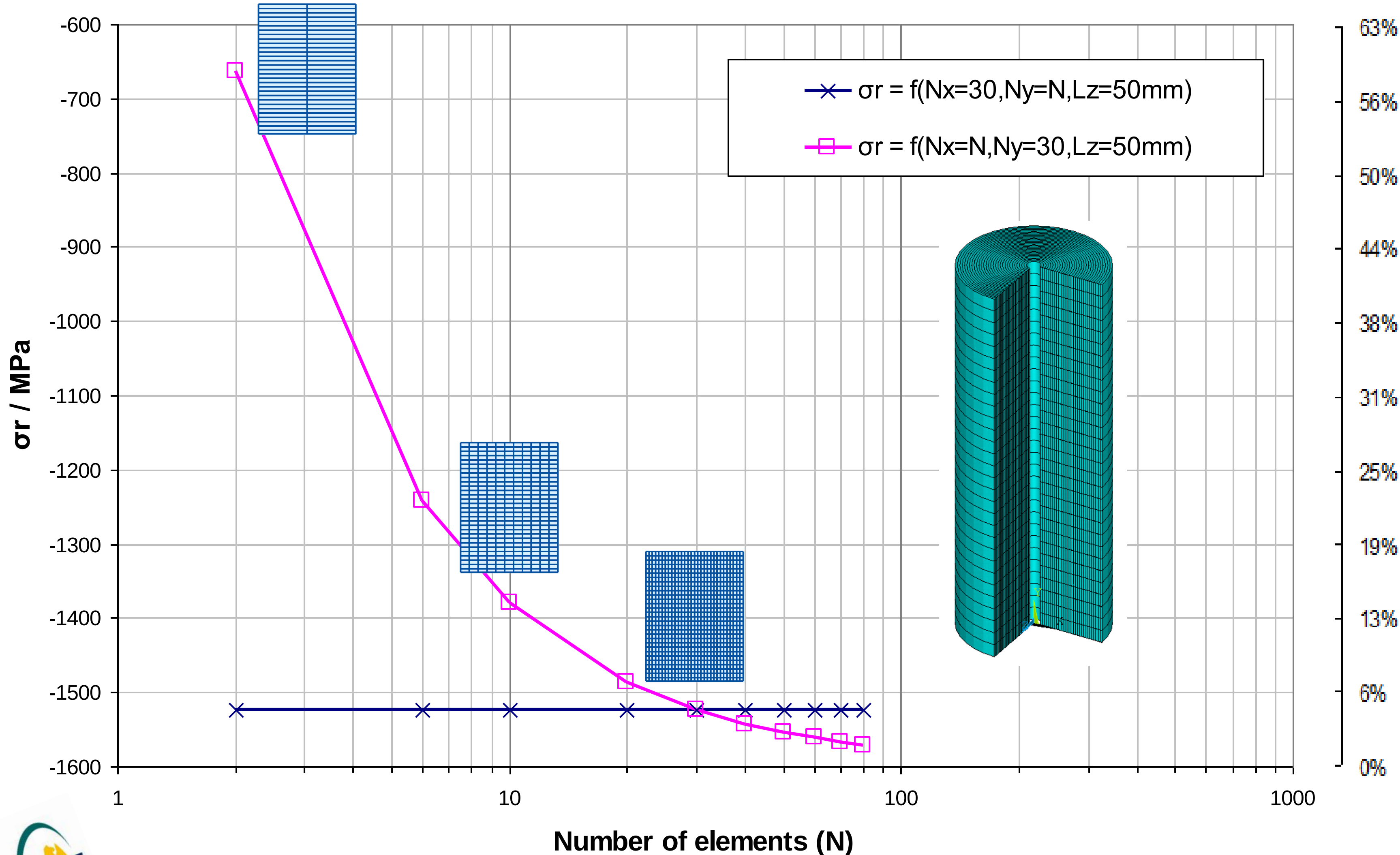
Allows parametric study on meshing, on typology of elements, ...

* APDL : Ansys Parametric Design Language

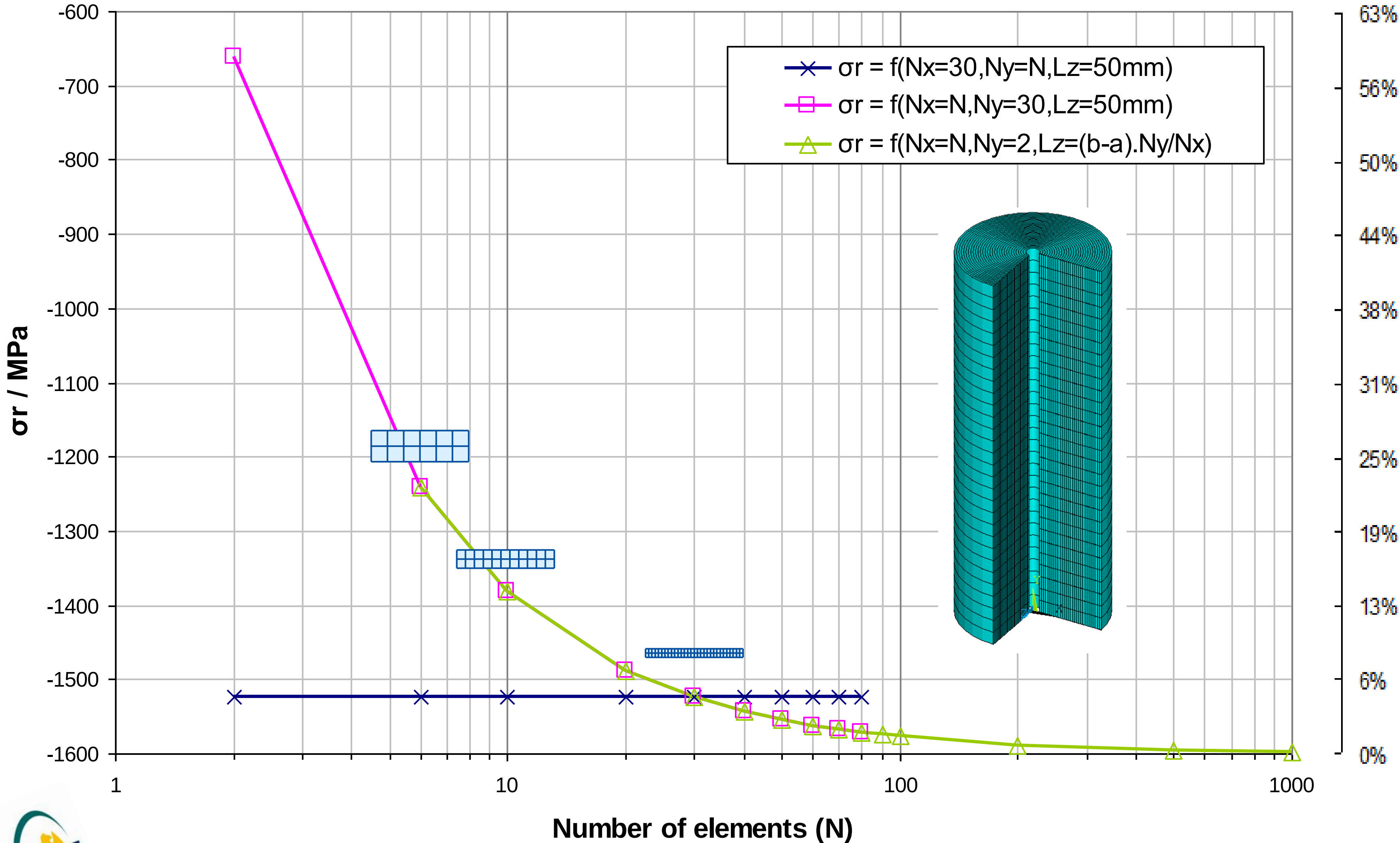
Numerical study



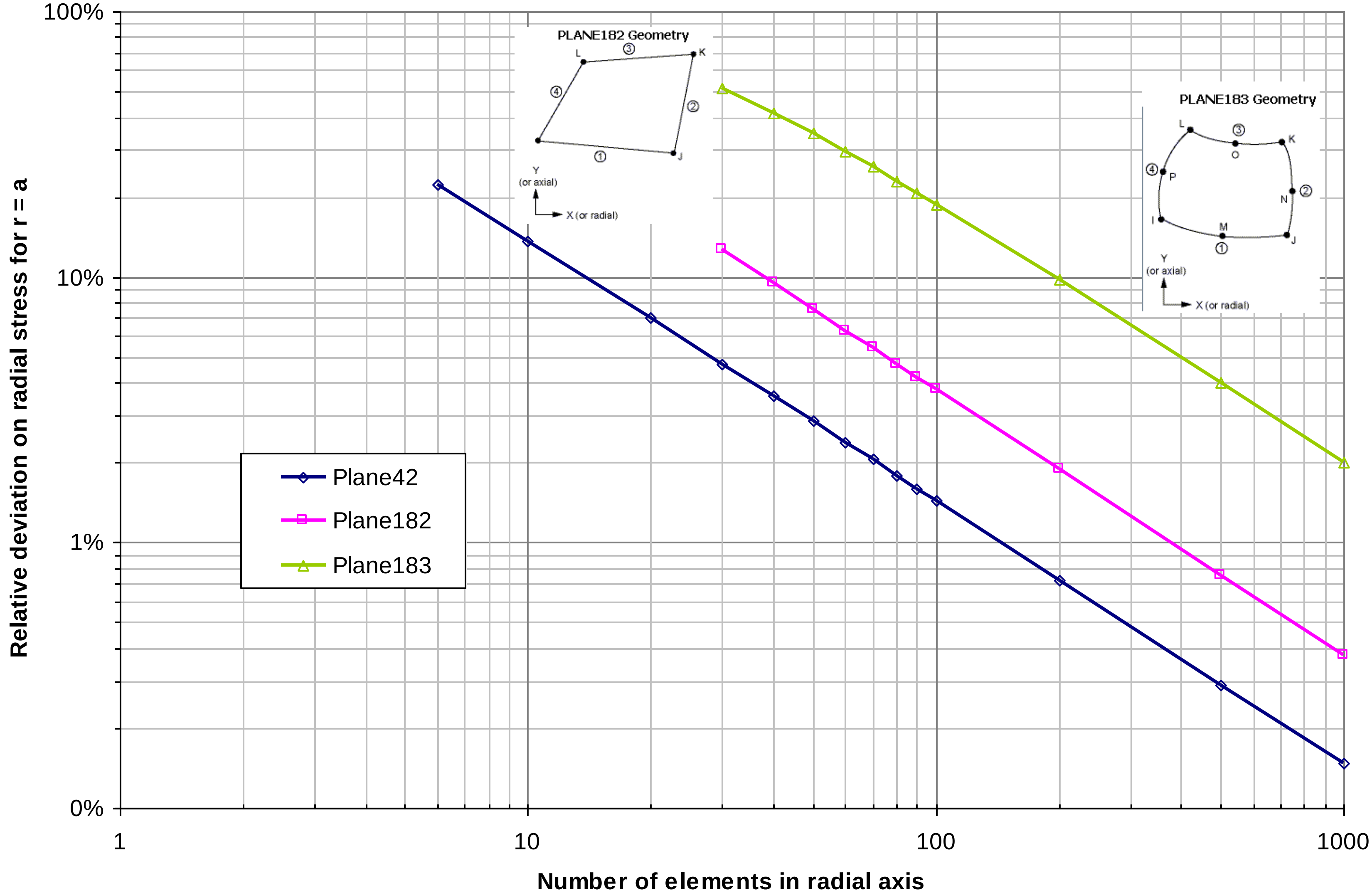
Numerical study



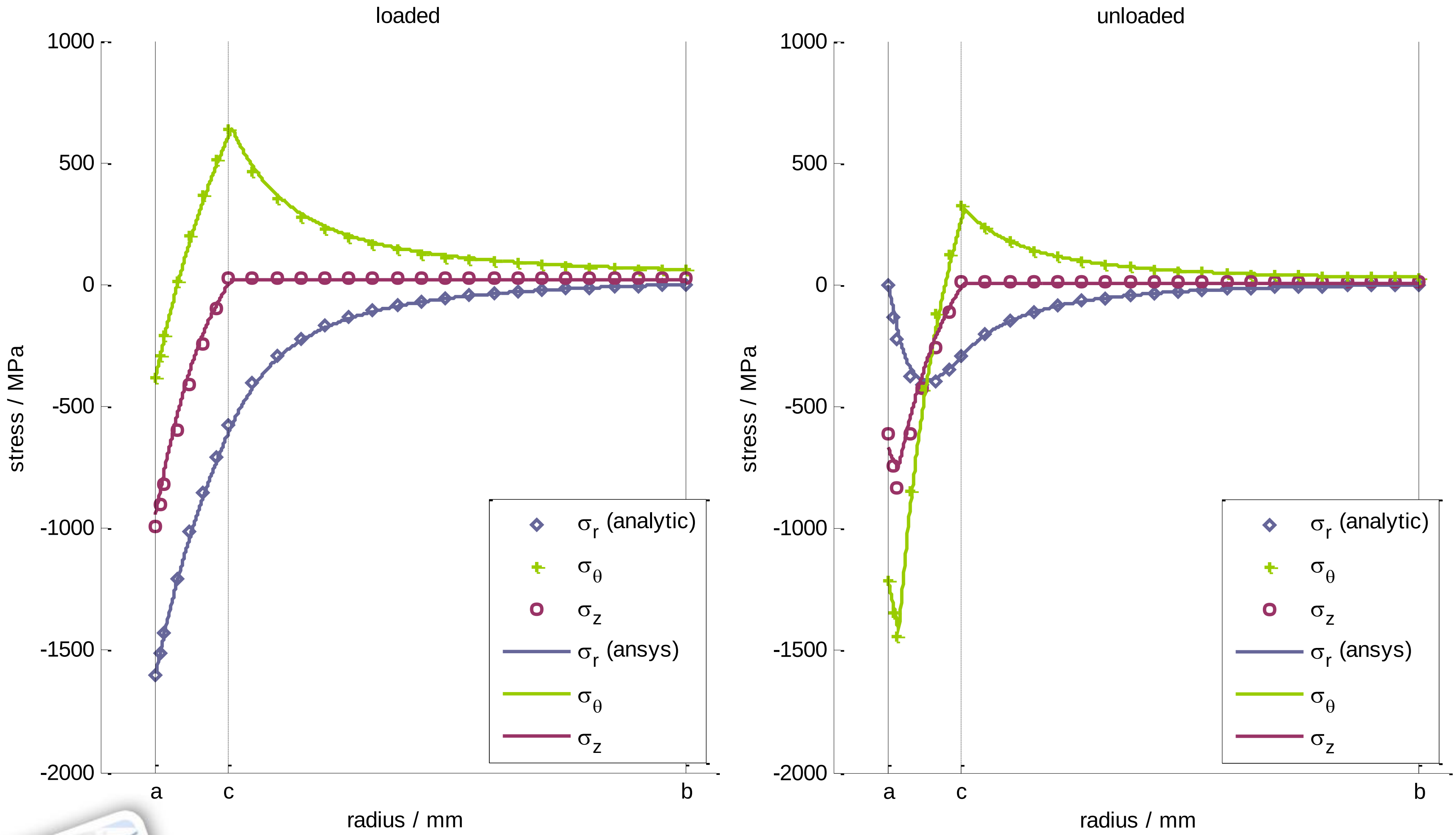
Numerical study

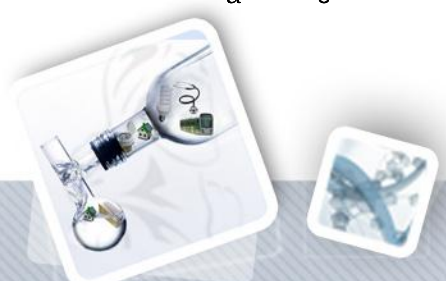


Numerical study



Numerical study





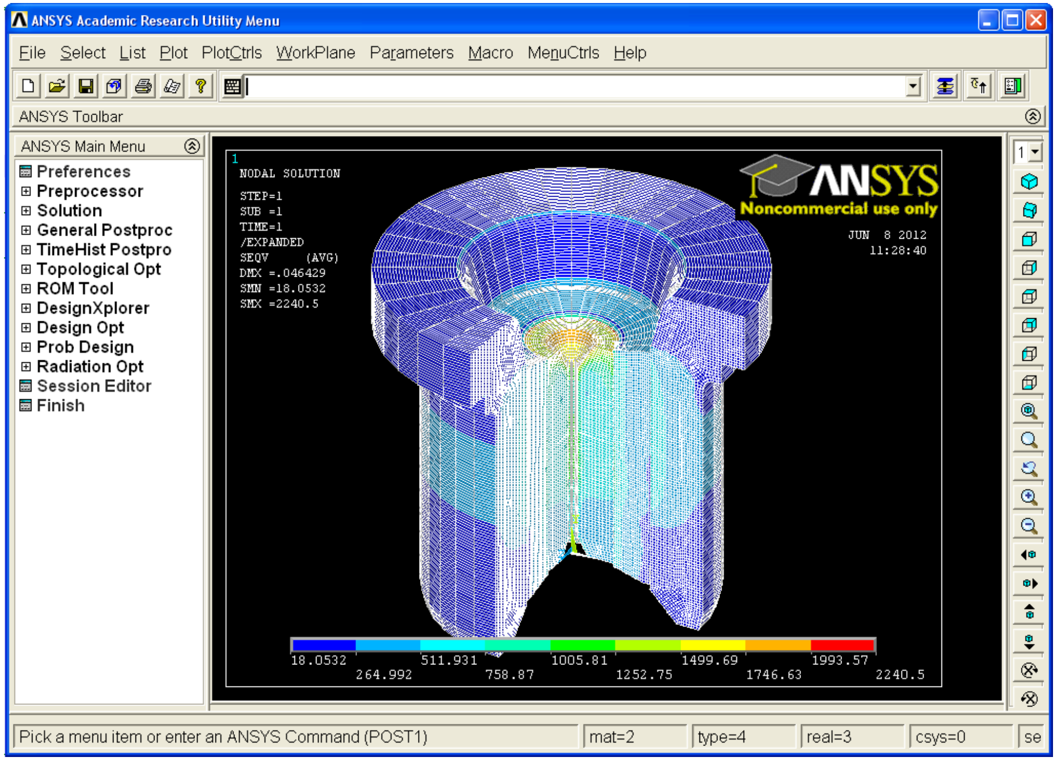
Conclusion

☑ No influence of the mesh density on the vertical axis.

☑ Refine the mesh along the radial axis emphasizing the edges

☑ Preferred elements type PLANE42

This informations will be useful to study more complex problems. Like stress analysis of high pressure piston-cylinder amplifier (Task 1.2)



Thank you for your attention.